

Exercise Sheet 9

Remember to specify your name, the number of your group and the name of the assistants in your group on the sheet that you hand in.

Question 1 (4 points)

Figure 1(a) illustrates an H_2O molecule. The O-H bond length is 0.96 \AA and the H-O-H bonds make an angle of 104° . Calculate the moment of inertia of the molecule (assume the atoms are points) about an axis passing through the center of the oxygen atom perpendicular to the plane of the molecule (figure 1(b)). Repeat the calculation when the axis passing through the oxygen atom is in the plane of the molecule, bisecting the H-O-H bonds (figure 1(c)).

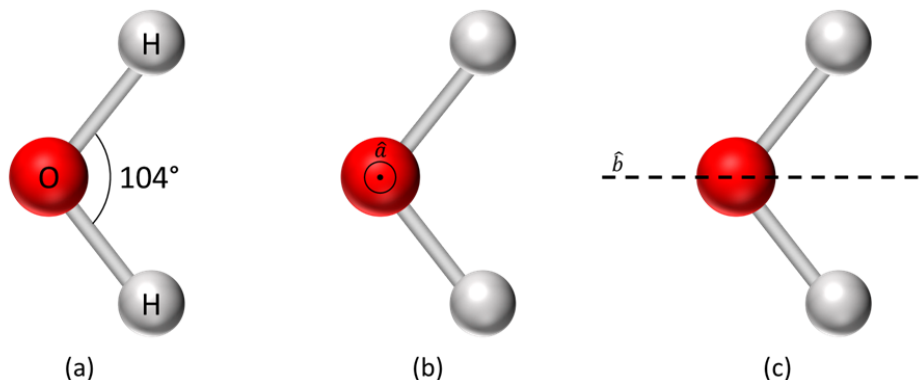


Figure 1: Schematic of question 1.

Question 2 (3 points)

On an inclined plane, a cylinder and a sphere with the same radius roll down, starting from rest at the same height. Which of the two objects reaches the bottom of the inclined plane first? When passing from the bottom of the inclined plane, what is the ratio between the speed of the cylinder and that of the sphere?

Question 3 (3 points)

A projectile of mass $m = 10 \text{ g}$ has an initial velocity $v_{01} = 60 \text{ m/s}$. It strikes a sandbag of mass $M = 2 \text{ kg}$, which is initially at rest ($v_{02} = 0$), and sticks into it. How fast does the sandbag recoil? How much energy is dissipated in the collision?

Problem 1 (10 points)

Two ivory spheres P_1 and P_2 of respective masses $m_1 = m$ and $m_2 = 2m$, are hanging on wires of length l each, so as to form two adjacent simple pendulums. Leaving mass m_2 stationary in its initial position ($\theta_{02} = 0$; $v_{02} = 0$), the mass m_1 is brought to form an angle $\theta_{01} = \theta_0 = 45^\circ$ with the vertical, and then released (figure 2).

- (a) What angles θ_1 and θ_2 of maximum elongation do the two pendulums reach after the collision? Assume the collision to be elastic.

Consider now the two pendulums as made of different materials. They are made to collide from the same initial conditions specified before ($\theta_1 = 45^\circ$; $v_{01} = 0$; $\theta_{02} = v_{02} = 0$). The impact is this time inelastic and frontal. The mass m_1 recoils after the impact with maximum elongation $\theta_1 = 8^\circ$.

- (b) What is the coefficient of restitution of the impact ε ?
- (c) What is the maximum elongation of mass m_2 ?

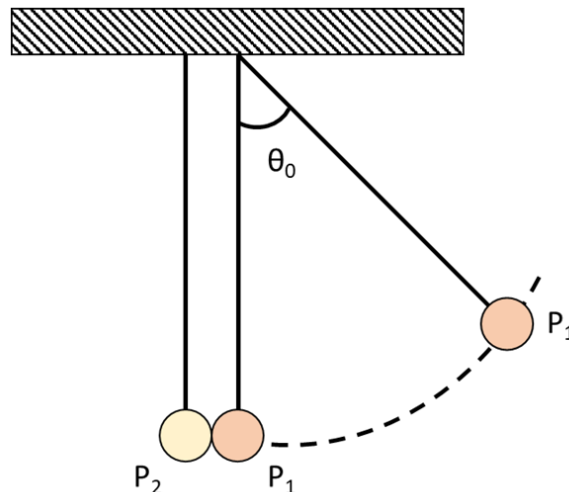


Figure 2: Schematic of problem 1.

Problem 2 (10 points)

A yo-yo is made of two solid cylindrical disks, each of mass 5×10^{-2} kg and diameter 75×10^{-3} m, joined by a (concentric) thin solid cylindrical hub of mass 5×10^{-3} kg and diameter 13×10^{-3} m. Evaluate:

- (a) the moment of inertia of one disk with respect to the axis orthogonal to the plane of the disk passing through the center of mass (ignore the rest of the pieces).
Hint: imagine dividing the disk in a series of circular crowns, of which the generic one has radius r and thickness dr .
- (b) the moment of inertia of one disk with respect to the axis \hat{a} , orthogonal to the disk and passing for the peripheral point A, where point A is located at a distance corresponding to the radius, from the center of the disk (ignore the rest of the pieces).

- (c) the linear speed of the yo-yo just before it reaches the end of its 1 m long string, if it is released from rest.
- (d) What fraction of its kinetic energy is rotational?