## Exercise Sheet 8

Remember to specify your name, the number of your group and the name of the assistants in your group on the sheet that you hand in.

## Question 1 (4 points)

A uniform circular plate of radius 2 R and center O , has a circular hole of radius R and center O' cut out of it. The distance between O and O' corresponds to 0.8R (Figure 1). What is the position of the center of mass of the plate?


Figure 1: Schematic of question 1.

## Question 2 (3 points)

A wedge of mass $\mathrm{M}=2 \mathrm{~kg}$, whose section is defined by a quarter of a circle of radius $\mathrm{R}=59 \mathrm{~cm}$ is on a horizontal plane without friction. A body of mass $\mathrm{m}=0.5 \mathrm{~kg}$ moves upward along the wedge with a certain velocity (Figure 2). Which minimum initial velocity needs the body to reach a height corresponding to $\mathrm{R} / 2$ ?


Figure 2: Schematic of question 2.

## Question 3 (3 points)

A train, consisting of a traction unit and three wagons, has an initial acceleration $\mathrm{a}=0.5 \mathrm{~m} / \mathrm{s}^{2}$. Assuming that, both the traction unit and the wagons have a mass $\mathrm{M}=20$ tons each, what is the force $\mathrm{F}_{1}$ that the traction unit exerts on the wagons? What is the resistant force $\mathrm{F}_{2}$ that the wagons exert on the traction unit? What force $\mathrm{F}_{3}$ does the first car exert on the second?

## Question 4 (2 points)

A small car and a heavy truck are both out of gas. The truck has twice the mass of the car. After you push both the car and the truck for the same amount of time with the same force, what can you say about the momentum and kinetic energy of the car and the truck? Ignore friction.
(a) They have the same momentum and the same kinetic energy
(b) The car has more momentum and more kinetic energy than the truck
(c) The truck has more momentum and more kinetic energy than the car
(d) They have the same momentum, but the car has more kinetic energy than the truck
(e) They have the same kinetic energy, but the truck has more momentum than the car

Choose now the correct answer (a) - (e) if you assume that you push both the car and the truck for the same distance with the same force.

Justify your answers.

## Question 5 (4 points)

A simple pendulum with mass $\mathrm{m}=0.5 \mathrm{~kg}$ and length $\mathrm{l}=60 \mathrm{~cm}$ is initially stationary in the stable equilibrium position. After the application of a horizontal impulse, the momentum of the pendulum in its lowest position becomes $\mathrm{p}=10 \mathrm{~N} \cdot \mathrm{~s}$. Calculate the constrain force of the wire at the point of maximum height.

## Question 6 (4 points)

A body rests on a horizontal platform that rotates about a vertical axis with variable angular velocity. The distance between the body and the point of intersection of the rotation axis with the rotating plane is $\mathrm{d}=0.1 \mathrm{~m}$. When the angular velocity reaches the value $\omega_{0}=4 \mathrm{rad} / \mathrm{s}$, the body, initially stationary, starts to move. Calculate the coefficient of static friction between body and platform.

## Question 7 (4 points)

The potential energy for a conservative force field is represented in a Cartesian reference frame as $U(x, y, z)=-x^{2}-x y+z^{2}$. Which is the angle between the force and the horizontal x -axis in correspondence of the point of coordinates $\mathrm{P}(1,0,0)$.

## Question 8 (3 points)

A bicyclist coasts down a $6^{\circ}$ hill at a steady speed of $4 \mathrm{~m} / \mathrm{s}$. Assuming a total mass of 75 kg (bicycle plus rider), what must be the cyclist's power to climb the same hill at the same speed?
Hint: the friction force when going uphill is the same magnitude as when going downhill.

## Question 9 (3 points)

Figure 3 shows the position vs time plot for two bicycles, A and B.
(a) Identify any instant at which the two bicycles have the same velocity.
(b) Which bicycle has the larger acceleration?
(c) At which instant(s) are the bicycles passing each other? Which bicycle is passing the other?
(d) Which bicycle has the larger instantaneous velocity?
(e) Which bicycle has the larger average velocity?


Figure 3: Schematic of question 9.

## Problem 1 (10 points)

A massless spring with spring constant $\mathrm{k}=10 \mathrm{~N} / \mathrm{m}$ is placed between a block of mass $\mathrm{M}_{1}=2 \mathrm{~kg}$ and a block of mass $\mathrm{M}_{2}=3 \mathrm{M}_{1}$. The blocks are initially at rest on a frictionless surface and they are held together so that the spring between them is compressed by an amount $\Delta \mathrm{x}=24 \mathrm{~cm}$ from its equilibrium length. The blocks are then released, and the spring pushes them off in opposite directions.
(a) Is there a net external force on the system before release?
(b) Calculate the elastic energy of the spring in the initial and final configurations. Consider the final configuration as the moment in which the spring has maximum elongation.
(c) Calculate the speeds of the two blocks when they detach from the spring.
(d) Calculate the kinetic energy of the system in the initial and final configurations. Consider the final configuration as the moment in which the spring has maximum elongation.
(e) Describe the motion of the CM of this system. Ignore the mass of the spring.

## Problem 2 (10 points)

Consider the system shown in Figure 4. The point mass A ( $\mathrm{m}_{A}=0.2 \mathrm{~kg}$ ), moves along a circular path of radius $\mathrm{R}_{A}=20 \mathrm{~cm}$, with a constant angular velocity $\omega=2 \mathrm{rad} / \mathrm{s}$, and at the time instant $\mathrm{t}=0$, it occupies the position of coordinates $\left(0, \mathrm{R}_{A}\right)$. The point mass B $\left(\mathrm{m}_{B}=0.3 \mathrm{~kg}\right)$, follows a circular path of radius $\mathrm{R}_{B}=12 \mathrm{~cm}$, with the same angular velocity of $A$, and at the time instant $t=0$, it occupies the position of coordinates $\left(-\mathrm{R}_{B}, 0\right)$. Evaluate the acceleration of the center of mass of the system.


Figure 4: Schematic of problem 2.

## Problem 3 (10 points)

An elevator cable breaks when a 925 kg elevator is 28.5 m above the top of a huge spring at the bottom of the shaft. Calculate
(a) the work done by gravity on the elevator before it hits the spring.
(b) the speed of the elevator just before striking the spring.
(c) the amount the spring compresses (Hint: here work is done by both the spring and gravity).

