## Exercise Sheet 10

Remember to specify your name, the number of your group and the name of the assistants in your group on the sheet that you hand in.

## Question 1 (3 points)

A cylindrical vessel of radius $\mathrm{R}=30 \mathrm{~cm}$ and containing oil (density $\rho=0.9 \mathrm{~g} / \mathrm{cm}^{3}$ ), rotates with an angular velocity $\omega=12 \mathrm{rad} / \mathrm{s}$ around its vertical axis $c$. A pressure gauge M , placed at the bottom of the vessel, as shown in figure 1 , measures a pressure $\mathrm{p}_{B}=1.1$ atm , while $\mathrm{p}_{0}=\mathrm{p}_{A}=1 \mathrm{~atm}$. You can consider the oil to be at rest relative to the vessel. Evaluate the height $\mathrm{h}_{A}$ of the oil on the axis of the cylinder.


Figure 1: Schematic of question 1.

## Question 2 (2 points)

The base of an insect's leg is approximately spherical in shape, with a radius of about $3 \times 10^{-5} \mathrm{~m}$. The insect's mass is 0.016 g and it's supported equally by its six legs. Would you expect the six-legged insect to remain on top of the water $(\sigma=72 \mathrm{mN} / \mathrm{m})$ ? Why?

## Question 3 (2 points)

A body is in equilibrium in the separation zone between two non-miscible liquids of density $\rho_{1}$ and $\rho_{2}$ respectively, with a fraction $\mathrm{f}_{2}$ of its total volume immersed in liquid 2 . What is the density of the body?

## Question 4 (3 points)

A U-shaped pipe with a constant cross-section, open at the ends towards the atmosphere, contains homogeneous liquid over a section of total length 1 (figure 2). In the initial situation a valve R keeps the liquid in an asymmetrical configuration, in which the difference in height between the free surfaces $A$ and $B$ is $h$. At a certain instant the valve $R$ is opened and the liquid begins to oscillate. Assuming the liquid is perfect, calculate the period of oscillation and the maximum velocity of the liquid with respect to the pipe.


Figure 2: Schematic of question 4.

## Problem 1 (10 points)

Water flows inside a tube, free of friction (figure 3) through the circular cross-section of area $S_{1}$, radius $R_{1}$ at height $h$. The velocity of the particles of the water at $S_{1}$ is $v_{1}$. The water exits from the tube through the circular cross-section $S_{2}$ (end of the tube) into the atmosphere. Evaluate:
(a) the dimensions of the product $\mathrm{v}_{1} \cdot \mathrm{~S}_{1}$ and hence state the units in SI
(b) the velocity $\mathrm{v}_{2}$ of the water at the exit from $\mathrm{S}_{2}$
(c) the pressure $P_{1}$ of the water in $S_{1}$

Data: $\mathrm{R}_{1}=2 \mathrm{~cm} ; \mathrm{R}_{2}=0.8 \mathrm{~cm} ; \mathrm{h}=3 \mathrm{~m} ; \mathrm{P}_{2}=1$ atmospheric pressure $=1.013 \times 10^{5}$ Pa; $\mathrm{v}_{1}=2 \mathrm{~m} / \mathrm{s} ; \rho=10^{3} \mathrm{~kg} / \mathrm{m}^{3}$


Figure 3: Schematic of problem 1.
Imagine now that the pipe shown in figure 3 is horizontal $(h=0)$.
(d) Calculate the mass of liquid passing through a section of pipe in a unit of time.

## Problem 2 (10 points)

Consider a pipeline with a diameter of 50 cm filled by oil. The Reynolds number is 1200 .
(a) Is the flow laminar or turbulent?

Evaluate
(b) the critical velocity of the fluid
(c) the flow rate for the value of the flow velocity found in (a)
(d) the pressure gradient per unit length for the value of the flow velocity found in (a)
(e) the power per unit length that must be expended to maintain the flow rate

Data: $\eta=0.7 \mathrm{~kg} / \mathrm{ms}, \rho=900 \mathrm{~kg} / \mathrm{m}^{3}, \mathrm{l}=1 \mathrm{~km}$

