Exercise Sheet 10

Remember to specify your name, the number of your group and the name of the assistants in your group on the sheet that you hand in.

Question 1 (3 points)

A cylindrical vessel of radius R = 30 cm and containing oil (density $\rho = 0.9$ g/cm³), rotates with an angular velocity $\omega = 12$ rad/s around its vertical axis c. A pressure gauge M, placed at the bottom of the vessel, as shown in figure 1, measures a pressure $p_B = 1.1$ atm, while $p_0 = p_A = 1$ atm. You can consider the oil to be at rest relative to the vessel. Evaluate the height h_A of the oil on the axis of the cylinder.



Figure 1: Schematic of question 1.

Question 2 (2 points)

The base of an insect's leg is approximately spherical in shape, with a radius of about 3×10^{-5} m. The insect's mass is 0.016 g and it's supported equally by its six legs. Would you expect the six-legged insect to remain on top of the water ($\sigma = 72 \text{ mN/m}$)? Why?

Question 3 (2 points)

A body is in equilibrium in the separation zone between two non-miscible liquids of density ρ_1 and ρ_2 respectively, with a fraction f_2 of its total volume immersed in liquid 2. What is the density of the body?

Question 4 (3 points)

A U-shaped pipe with a constant cross-section, open at the ends towards the atmosphere, contains homogeneous liquid over a section of total length l (figure 2). In the initial situation a valve R keeps the liquid in an asymmetrical configuration, in which the difference in height between the free surfaces A and B is h. At a certain instant the valve R is opened and the liquid begins to oscillate. Assuming the liquid is perfect, calculate the period of oscillation and the maximum velocity of the liquid with respect to the pipe.



Figure 2: Schematic of question 4.

Problem 1 (10 points)

Water flows inside a tube, free of friction (figure 3) through the circular cross-section of area S_1 , radius R_1 at height h. The velocity of the particles of the water at S_1 is v_1 . The water exits from the tube through the circular cross-section S_2 (end of the tube) into the atmosphere. Evaluate:

- (a) the dimensions of the product $v_1 \cdot S_1$ and hence state the units in SI
- (b) the velocity v_2 of the water at the exit from S_2
- (c) the pressure P_1 of the water in S_1

Data: R_1 = 2 cm; R_2 = 0.8 cm; h = 3 m; P_2 = 1 atmospheric pressure = 1.013 \times 10^5 Pa; v_1 = 2 m/s; ρ = 10^3 kg/m^3



Figure 3: Schematic of problem 1.

Imagine now that the pipe shown in figure 3 is horizontal (h = 0).

(d) Calculate the mass of liquid passing through a section of pipe in a unit of time.

Problem 2 (10 points)

Consider a pipeline with a diameter of 50 cm filled by oil. The Reynolds number is 1200.

(a) Is the flow laminar or turbulent?

Evaluate

- (b) the critical velocity of the fluid
- (c) the flow rate for the value of the flow velocity found in (a)
- (d) the pressure gradient per unit length for the value of the flow velocity found in (a)
- (e) the power per unit length that must be expended to maintain the flow rate

Data: $\eta=0.7~{\rm kg/ms},\,\rho=900~{\rm kg/m^3},\,l=1~{\rm km}$